Radar Beacon Tracking with Downlinked Heading and Airspeed

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Position, ground velocity, and the wind are estimated using flight data acquired by the Discrete Address Beacon System (DABS). The measurements consist of radar range and bearing plus telemetered airspeed and heading sampled every 4 s. The data are used to determine statistics for the random errors in range and bearing, the random compass and airspeed errors, and the flight technical error in holding heading and airspeed. The random errors were found to be small in comparison to airspeed and heading-dependent compass biases. After correction for biases a fixed-gain filter provided wind estimates with an uncertainty of 2 knots (1 σ) over a bandwidth of 0.05 Hz. The air-derived data were effective in improving tracking during vehicle accelerations or when a component of the DABS data was missing or inaccurate. Deterioration of the quality of measured data was detectable by monitoring the measurement residuals. The results have implications for systems which attempt to do large-area wind mapping with radar and telemetered air data.

Nomenclature

E = estimate error covariance matrix
H = measurement matrix
L = lead aircraft heading or airspeed
$\mathbf{m}$ = measurement vector
N = measurement error covariance matrix
p = position component
R = residual covariance matrix
t = time
TAS = true airspeed component
$\mathbf{v}$ = true airspeed vector
W = wing aircraft heading or airspeed
w = wind component
$\mathbf{x}$ = state vector
x = north geographic coordinate
y = east geographic coordinate
Δt = time between scans
σ = standard deviation
Φ = state transition matrix

Superscripts

$'$ = value after incorporation of measurement
$\hat{}$ = estimated value
$\tilde{}$ = measured value
T = transpose of matrix
-1 = inverse of matrix

Introduction

THE purpose of this paper is to examine the accuracy of position, velocity, and wind estimation obtainable by combining radar surveillance position data with aircraft airspeed and heading data. The surveillance data were supplied by the Discrete Address Beacon System (DABS) sensor[1] that was developed for incorporation of an Intermittent Positive Control (IPC) collision avoidance concept.[2] IPC is an automatic ground-based collision avoidance system that requires a data link to transmit appropriate collision avoidance and traffic advisory information to an affected aircraft. The airspeed and heading data were part of the RAS (Readout of Aircraft State) information sent down on the DABS data link.[3] RAS data also included altitude, outside air temperature, bank angle, pitch angle, and rate of climb. RAS was incorporated for the purpose of IPC testing; it is not part of the IPC system.

The primary reason for combining airspeed and heading data with radar beacon derived position is that it allows an estimate of the wind. Secondary advantages are that it improves velocity estimation during acceleration, it can improve position estimation when the DABS data are poor or missing, and it can provide a monitor of the DABS accuracy in certain instances. Knowledge of the wind will be essential for advanced air traffic control concepts. Automated conflict resolution, metering, spacing, and flow control all require vectoring commands from the ground to the aircraft. The commands include heading, altitude, and airspeed. In order to compute the commands it is necessary to know the wind. Airspeed and heading can be inferred from vectoring commands sent up to the aircraft if they are not available by downlink. The difference between the commanded heading or airspeed and the actual heading or airspeed is defined as flight technical error. Flight technical error has the same importance as instrument sensor error when commanded heading or airspeed is used in place of downlinked measurements. It is also possible to make similar estimates on board the aircraft by uplinking the DABS position data. In each case the accuracy of the wind estimate will be strongly dependent upon the accuracy of the heading and airspeed sensors and, in some cases, upon the flight technical error. Experiments were conducted to determine the magnitude of these error sources. The data were collected during flight test of the IPC system. A datum point was available every 4 s, the rotation period of the DABS antenna. A simple, fixed-gain filter was used to combine the data to make the wind estimate and improve the position estimate.

The Estimator

The estimator combines the dead-reckoned position obtained from RAS information (heading and true airspeed) together with DABS surveillance measurements to form a new estimate of position and the wind after each scan. The estimate would be optimum if the assumed model is adequate, the measurements are unbiased, and the assumed error covariances are correct. In operation, the error covariances are assumed to be known constants so that the estimator acts

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like a fixed-gain filter that is rigorously optimum only in the steady state. The state, $x(t)$, is given by

$$x(t)=\begin{bmatrix} p_y \\ p_x \\ w_y \\ w_x \end{bmatrix} \tag{1}$$

The state estimate $\hat{x}$ is propagated forward between measurements by dead reckoning

$$\hat{x}(t+\Delta t)=\Phi(t,t+\Delta t)\hat{x}(t)+\left[\frac{v(t)+v(t+\Delta t)}{2}\right]\Delta t \tag{2}$$

where

$$\Phi(t,t+\Delta t)=\begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$v(t)=\begin{bmatrix} \mathrm{TAS}_y \\ \mathrm{TAS}_x \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

where $\mathrm{TAS}_y = y$ component of telemetered (RAS) true airspeed at time t and $\mathrm{TAS}_x = x$ component of telemetered (RAS) true airspeed at time t. After each scan the new estimate of the state $\hat{x}'$ is obtained from the dead-reckoned estimate $\hat{x}$ and the new DABS measurement by

$$\hat{x}'=x+EH^T[HEH^T+N]^{-1}(\tilde{m}-\hat{m}) \tag{5}$$

where

$$\tilde{m}=\begin{bmatrix}\text{DABS range measurement} \\ \text{DABS azimuth measurement}\end{bmatrix}$$

$$\hat{m}=\begin{bmatrix}\text{range predicted by } \hat{x} \\ \text{azimuth predicted by } \hat{x}\end{bmatrix}$$

$$H\triangleq\frac{\partial m}{\partial x}=\text{measurement matrix}$$

For the assumed model

$$H=\begin{bmatrix} p_y/(p_x^2+p_y^2)^{1/2} & p_x/(p_x^2+p_y^2)^{1/2} & 0 & 0 \\ p_x/(p_x^2+p_y^2) & -p_y/(p_x^2+p_y^2) & 0 & 0 \end{bmatrix} \tag{6}$$

$$E\triangleq\overline{(\hat{x}-x)(\hat{x}-x)^T}=\text{estimate error covariance matrix}$$

E is assumed to be of the constant form

$$E=\begin{bmatrix} \sigma_y^2 & 0 & (\sqrt{2}/2)\sigma_y\sigma_{w_y} & 0 \\ 0 & \sigma_x^2 & 0 & (\sqrt{2}/2)\sigma_x\sigma_{w_x} \\ (\sqrt{2}/2)\sigma_y\sigma_{w_y} & 0 & \sigma_{w_y}^2 & 0 \\ 0 & (\sqrt{2}/2)\sigma_x\sigma_{w_x} & 0 & \sigma_{w_x}^2 \end{bmatrix} \tag{7}$$

$N\triangleq\overline{(\tilde{m}-m)(\tilde{m}-m)^T}$ = measurement error covariance matrix.

For the assumed model for m

$$N=\begin{bmatrix}\sigma^2_{\text{range}} & 0 \\ 0 & \sigma^2_{\text{azimuth}}\end{bmatrix} \tag{8}$$

The vector residual r of the DABS measurement is given by

$$r=\tilde{m}-\hat{m} \tag{9}$$

The residual covariance matrix R is defined by

$$R\triangleq\overline{(\tilde{m}-\hat{m})(\tilde{}-\hat{m})^T} \tag{10}$$

and, if the filter is optimum,

$$R=N+HEH^T \tag{11}$$

The residuals can be monitored to detect instances when their magnitudes are not consistent with values predicted by Eq. (11).

A computer program to implement this model was written using a modified version of an IPC data analysis routine to read nine-track tapes which contained all flight data for each mission.[4] When a residual exceeds 3 σ of its associated measurement, it is flagged. If the analyst has some additional information (such as a pilot report that a turn was or was not initiated) he can tell which data source was unreliable. It is important to note that the values chosen for E and N determine the gains of the estimator. They may or may not be a good approximation of the existing statistics. Although the filter is optimum when E and N are correct, the residuals are not particularly sensitive to changes in the estimator gains. This was observed previously in an analytic study using this same filter[5] and was confirmed in the present case by experimenting with the data. The point is that the filter will work satisfactorily even when the assumed gains do not accurately reflect the existing statistics. The values used as estimator gains were $\sigma_{\text{range}}=30$ ft, $\sigma_{\text{azimuth}}=0.035$ deg, $\sigma_x=\sigma_y=100$ ft, and $\sigma_{w_x}=\sigma_{w_y}=2$ knots. The values for N were assumed from previous data.[6] The values of E were assumed by intuition. At these gains the filter reaches steady state after six scans (24 s). This was determined by the time it takes for the wind estimate to reach a steady state when the aircraft is on a steady heading, airspeed, and altitude. Consequently, the estimator acts as a wind sensor with a bandwidth of about 0.05 Hz. The slow response time can cause the estimator to be inaccurate during descent through wind gradient or while flying through wind shear.

Bias Errors in the RAS Data

Because of the form of the estimator bias errors in the RAS, sensors are absorbed into the wind estimate. An experiment was performed to provide controlled data to determine RAS airspeed and heading biases, to compute the wind that existed during the flight as well as to establish the magnitude of random errors. In the experiment two test aircraft were flying a set of headings in close formation. Two hexagonal courses were flown, the second offset 30 deg from the first for a total of 12 headings. The pilots were instructed to hold the same constant altitude and airspeed on each heading, and to hold each heading as steadily as possible. Flying the headings in formation helped to separate instrument error from flight technical error. Balloon soundings from the National Weather Service at Chatham, Mass., were recorded to give a qualitative estimate of the wind and air temperature in the area during the experiment. It was assumed that temperature and altimeter errors would be small and have a negligible effect on true airspeed calculations. At the airspeed and altitude flown an error of 10°C results in an airspeed error of about 2 knots.

Table 1 Statistics for aircraft 101

Leg	Observed	Heading deg magnetic Estimated	Bias	σ^a
1	286	290	−4	1.3
2	346	352	−6	2.6
3	046	049	−3	2.2
4	107	112	−5	1.2
5	171	172	−1	1.7
6	231	230	+1	1.4
7	317	321	−4	2.5
8	017	017	0	1.6
9	077	076	+1	1.7
10	140	143	−3	1.7
11	202	199	+3	1.8
12	259	259	0	2.2

Leg	Observed, knots	Indicated airspeed Estimated, knots	Bias	σ^b
1	113	104	+9	2.8
2	113	104	+9	3.8
3	112	103	+9	3.8
4	111	102	+9	2.6
5	111	102	+9	2.4
6	111	102	+9	2.8
7	112	103	+9	2.5
8	109	100	+9	3.2
9	108	99	+9	3.5
10	111	102	+9	2.7
11	112	103	+9	2.7
12	112	103	+9	2.7

[a] $\sigma_{rms} = 1.8$ deg. [b] $\sigma_{rms} = 3.0$ knots.

The first step was to determine the best wind estimate based on the average wind measured by the filter on each leg of the flight. For each leg the along-track component of the wind is in error due to the airspeed bias, and the cross-track component of the wind is in error due to the compass bias. Consequently, the along-track measured wind components were used to estimate the two components of the true wind plus the airspeed bias. A minimum of three noncolinear, along-track wind measurements are necessary to estimate the two wind components plus airspeed bias. With more than three measurements the estimate can be improved and the statistics of the measurement observed. Finally, the compass bias errors for each heading were determined using the estimated wind and the airspeed corrected for bias error. The process assumes that the average wind and the airspeed bias remain constant.

The experiment was flown April 28, 1976. The 12 measurements of the wind resulted in an estimate of 313 deg at 21 knots with an uncertainty of 2 knots. By way of comparison, the Chatham 0700 wind differed from the estimate by 4 knots. The Chatham 1900 wind differed by 3 knots. A Flight Service Station forecast of the wind differed by 6 knots. This analysis identified an airspeed bias error of +9 knots for Aircraft 101 and −10 knots for Aircraft 552. Adjusting the telemetered airspeed data to account for this bias produced numbers which agreed with the pilots' reports of the airspeed indications on their panel instruments. The results of the experiment are noted in Tables 1 and 2. The observed value is the mean of the RAS reports for the leg. Sigma is the standard deviation of the RAS reports for the leg. The estimated airspeed is the observed airspeed corrected for the airspeed bias determined by assuming the average wind to be constant over all legs. The estimated heading is determined by subtracting the estimated wind vector from the observed ground velocity vector and taking its direction relative to magnetic north. Heading bias is the difference between observed and estimated heading on each leg.

The large heading-dependent compass biases are considered to be typical of general aviation aircraft. The aircraft are not swung on a compass rose regularly and compass calibration is known to depend upon aircraft configuration. A random sample of compass calibration cards in light aircraft confirmed that large heading-dependent compass biases do exist. The airspeed biases were considered large even though pitot static systems are not regularly calibrated in light aircraft.

Random Error in RAS Data

For a single aircraft there are two contributions to the random variations in measured heading or airspeed. One is the actual random deviation of the aircraft from the desired heading or airspeed. This component is called flight technical error. The other component is the random error of the sensor. Assuming independent random variables, it is possible to separate these two by observing the difference between the indicated headings (or airspeeds) of two aircraft flying in formation. The pilot of the lead aircraft is attempting to hold a constant heading and airspeed. The wing man is attempting to match the heading and airspeed of the leader.

Consequently, the random variations from the sensors in the wing aircraft include the flight technical error of the lead aircraft, while the difference between the sensors in the two aircraft does not contain the flight technical error of the lead aircraft. Analytically, where L and W denote lead and wing aircraft, respectively,

$$\sigma^2_{\tilde{L}} = \sigma^2_L + \sigma^2_{(\tilde{L}-L)} \tag{12}$$

$$\sigma^2_{\tilde{W}} = \sigma^2_L + \sigma^2_{(L-W)} + \sigma^2_{(\tilde{W}-W)} \tag{13}$$

$$\sigma^2_{(\tilde{L}-\tilde{W})} = \sigma^2_{(\tilde{L}-L)} + \sigma^2_{(L-W)} + \sigma^2_{(\tilde{W}-W)} \tag{14}$$

solving

$$\sigma^2_L = \tfrac{1}{2}\left[\sigma^2_{\tilde{L}} + \sigma^2_{\tilde{W}} - \sigma^2_{(\tilde{L}-\tilde{W})}\right] \tag{15}$$

$$\sigma^2_{(\tilde{L}-L)} = \tfrac{1}{2}\left[\sigma^2_{\tilde{L}} + \sigma^2_{(\tilde{L}-\tilde{W})} - \sigma^2_{\tilde{W}}\right] \tag{16}$$

$$\sigma^2_{(L-W)} + \sigma^2_{(\tilde{W}-W)} = \tfrac{1}{2}\left[\sigma^2_{\tilde{W}} + \sigma^2_{(\tilde{L}-\tilde{W})} - \sigma^2_{\tilde{L}}\right] \tag{17}$$

Table 2 Statistics for aircraft 552

Leg	Observed	Heading, deg magnetic Estimated	Bias	σ^a
1	286	291	−5	1.0
2	347	352	−5	2.4
3	046	050	−4	1.4
4	106	112	−6	1.0
5	166	172	−6	1.4
6	226	230	−4	1.4
7	317	321	−4	1.4
8	016	017	−1	1.2
9	076	076	0	1.6
10	137	143	−6	1.5
11	196	200	−4	1.0
12	255	259	−4	1.6

Leg	Observed, knots	Indicated airspeed Estimated, knots	Bias	σ^b
1	96	106	−10	2.0
2	97	107	−10	2.8
3	95	105	−10	2.5
4	93	103	−10	1.6
5	93	103	−10	1.8
6	93	103	−10	2.2
7	96	106	−10	2.3
8	94	104	−10	2.4
9	94	104	−10	2.2
10	95	105	−10	2.6
11	95	105	−10	1.4
12	95	105	−10	2.3

[a] $\sigma_{rms} = 1.5$ deg. [b] $\sigma_{rms} = 2.2$ knots.

The compass error of the wing aircraft and the flight technical error of the wing man are not distinguishable without further assumption.

For heading: $\sigma_{\tilde{L}} = 1.5$ deg (the observed $\sigma_{\rm rms}$ for aircraft 552), $\sigma_{\tilde{W}} = 1.8$ deg (the observed $\sigma_{\rm rms}$ for aircraft 101), and $\sigma_{(\tilde{L}-\tilde{W})} = 1.2$ deg (σ of difference in reported heading); lead flight technical error $= \sigma_L = 1.4$ deg from Eq. (15), lead aircraft random compass error $= \sigma_{(\tilde{L}-L)} = 0.5$ deg from Eq. (16). The flight technical error is greater than the random compass error. Both are small relative to the heading dependent compass bias.

For airspeed: $\sigma_{\tilde{L}} = 2.2$ knots (the observed $\sigma_{\rm rms}$ for aircraft 552), $\sigma_{\tilde{W}} = 3.0$ knots (the observed $\sigma_{\rm rms}$ for aircraft 101), and $\sigma_{(\tilde{L}-\tilde{W})} = 2.7$ knots (σ of difference in reported airspeed); lead flight technical error $= \sigma_L = 1.8$ knots from Eq. (15); lead aircraft random airspeed error $= \sigma_{(\tilde{L}-L)} = 1.2$ knots from Eq. (16). The flight technical error is again greater than the random instrument error. Both are small relative to the airspeed bias errors.

It is interesting as an aside to estimate the wing man's flight technical error. If the wing aircraft is assumed to have the same random instrument error as the lead aircraft, then

$$\sigma^2_{(L-W)} = \sigma^2_{\tilde{W}} - \sigma^2_{\tilde{L}} \tag{18}$$

The wing man's heading flight technical error is 1 deg, and the airspeed flight technical error is 2 knots. Apparently the wing man can match the leader's heading better than the leader can hold heading, but the leader can hold airspeed better than the wing man can match the leader's airspeed.

DABS Residuals

The DABS residuals, $\tilde{m} - \hat{m}$, have typical rms values of 30 ft in range and 0.035 deg in bearing. Since both the measurement errors and the estimation errors contribute to the residuals, it is safe to estimate that the 1 σ DABS random errors are better than 30 ft and 0.035 deg, respectively. This is consistent with other measures of DABS accuracy.[6] By monitoring the residuals it is possible to detect anomalous behavior on the part of one of the sensors.

The capability of the filter to monitor DABS accuracy was explored through analysis of a portion of mission 6-35S, flown on Aug. 1, 1975. During this mission the aircraft flew through a region where diffraction was subsequently found to be the cause for severe DABS azimuth errors. Because of limited data, it was not possible to determine the RAS biases for this mission, therefore errors due to biases in heading and airspeed could contribute to the residuals after turns. Based on the magnitude of the residuals it appeared that significant deterioration in the DABS azimuth accuracy occurred at about scan 641 and returned to normal after scan 751. Ground track plots and range and azimuth residual plots of a 100-scan segment are shown in Figs. 1-3. Horizontal lines in Figs. 2 and 3 indicate 3 σ of the subject residual.

There are several reasons that the DABS azimuth measurement was thought to be in error. The primary one is that azimuth measurements are known to deteriorate in quality when the target is behind a tower, smokestack, or other obstruction that can cause signal diffraction.[7] There is a smokestack between the DABS antenna and the area in which the airplane was flying which would interfere with the signal at the combination of altitude and range which existed at the time. Also, the range residuals did not show the same increase

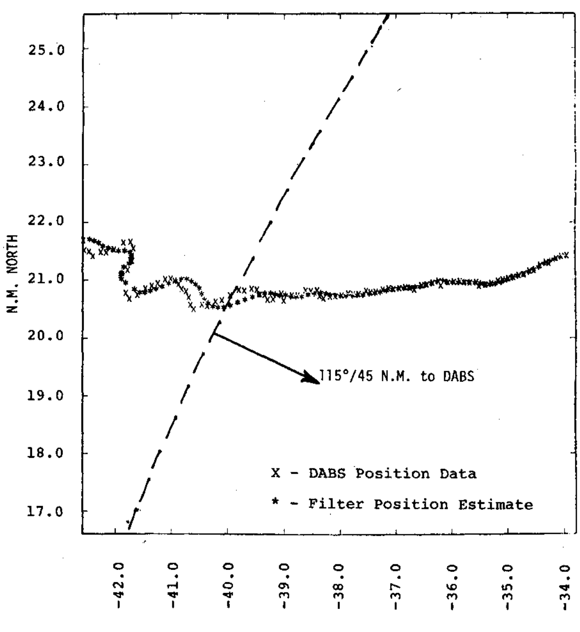

Fig. 1 DABS and filter position estimates: scans 697-795, mission 6-35-S.

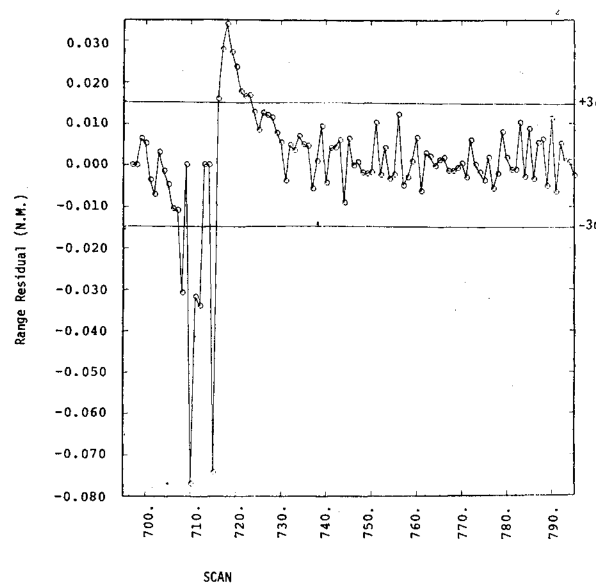

Fig. 2 Range residual vs scan, mission 6-35-S.

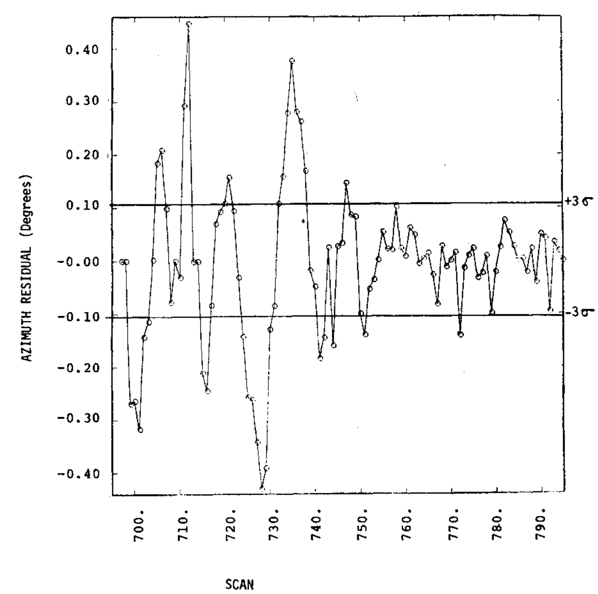

Fig. 3 Azimuth residual vs scan, mission 6-35-S.

as the azimuth residuals. Finally, it is not likely that the RAS heading and airspeed would be subject to 30-deg and 25-knot errors over the course of 2 or 3 min; particularly when they agree qualitatively with pilot reports and the DABS track prior to and after the affected area of the flight.

The large excursions in the range residuals in the region around scan 710 are due to missing data. Neither RAS nor DABS information was available to the filter for one or more scans. When the filter receives no data it uses the previous heading, airspeed, and wind values. The aircraft was in a turn at each of these points; thus, when the data resumed one or two scans later there was a large difference between the estimated range and the measured range. It was several scans before the filter's estimated range could be back to within 3σ. A significant benefit might be achieved by a model which better accounted for turns and missing data.

The variation of the azimuth residuals is almost sinusoidal during the anomalous behavior. It seems that in this instance the azimuth error does not behave like a random noise.

The range residuals plotted after scan 730 represent a typical output when all data sources are functioning normally. The azimuth residuals after scan 750 also represent a typical output. These areas of the plots behave the same way as other residual output from missions where no anomalies were present.

Conclusions

The following conclusions are drawn from the results of the investigation. (Accuracy is quoted in terms of $1\ \sigma$.) The observed measurement residuals for the DABS sensor were less than 30 ft in range and 0.035 deg in bearing. Random errors in the compass and airspeed indicators were found to be 0.5 deg and 1.2 knots, respectively. Flight technical errors for holding heading and airspeed were found to be 1.4 deg and 1.8 knots, respectively.

Heading-dependent compass biases as large as 6 deg and airspeed biases as large as 10 knots were observed. A simple fixed-gain wind estimator worked satisfactorily. The wind could be estimated to an accuracy of 2 knots over a 0.05-Hz bandwidth after removal of the bias errors.

The measurement residuals provide a good monitor of DABS accuracy. Deduction is necessary to isolate which component of the measurements has gone bad after the residuals identify that a problem exists.

The main advantage of heading data is that they improve tracking during turns. The heading-dependent compass biases must be determined before the heading data are useful for wind estimation. Airspeed data are more useful than heading for wind estimation. The airspeed bias and the wind components can be estimated from the radar and airspeed data alone on three nonparallel tracks. Airspeed data improve tracking during longitudinal accelerations. Heading and airspeed together improve position estimation when radar data are very poor or missing.

The results of the study can be useful in the design of a system which attempts to make a large-area wind map using heading or airspeed downlinked from many aircraft. The bias errors can be expected to be larger than the random errors for such a system. Differential heading or heading rate, which is easier to sense, may be just as useful as heading to assist the tracker during turns. The wind could be estimated from airspeed data alone so long as there are enough reporting aircraft on nonparallel tracks. The system should make an estimate of the airspeed bias of each reporting aircraft in addition to the common wind.

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